Response of thermal storage units to periodic operating conditions

G. Spiga* and M. Spiga†

This paper aims to provide an exact solution to the problem of sensible heat transfer and storage in fluid–solid systems. Periodically fluctuating operating conditions are considered in detail, and a simple solution procedure is proposed in order to obtain the fluid and solid temperature distributions for any periodic thermal energy supply. Some significant examples are solved explicitly, and the relevant results are reported and briefly discussed in order to examine directly the storage unit's capability of smoothing out time fluctuations of the power source.

Key words: *heat transfer, energy storage, fluid–solid systems*

Thermal energy storage offers a means of obtaining significant cost reductions in all problems involving both energy saving and the development of alternative energy sources. Significant existing or potential applications can be found in many general areas such as power generation and industrial or domestic utilization.

High-temperature sensible heat devices are in widespread industrial use, using the principle of heat transfer from a working fluid to a storage material, with subsequent energy recovery by reheating the fluid. Promising applications exist in gas turbine cycles (using solar, fossil, waste-heat sources), in superheated steam storage with nuclear or fossil-fuelled electric power plant, and in energy storage using compressed air with recovery of the heat of compression[1]. In solar electrical conversion the storing of large amounts of high-temperature energy is a key technology for successful exploitation of this energy source on a significant scale, permitting the intermittent solar energy to be used later upon demand[2].

In any storage system, identification of the optimal operating conditions requires an understanding not only of the energy supply, but also of how this energy is distributed within the store unit. In this type of situation, the energy distribution seems to vary with time; in many practical applications the temperature of the fluid entering the storage system is subjected to periodic variations, which take place repeatedly for the whole operational time. With this in view a method has been proposed recently[3,4] for estimating system performance using sinusoidal inputs.

The work reported here aimed to derive a general analytical solution for the transient response of a sensible heat storage system, referred to periodic operating conditions, such as those of most common industrial and domestic applications[5–8]. The mathematical treatment of thermal transfer processes is based on a two-phase model in which both fluid and solid preserve their own individuality, the solid phase being a porous medium where fluid flows in the void spaces. This is certainly more realistic than a one-phase model in which the store unit is approximated by a homogeneous medium with uniform properties. The fluid-to-solid heat capacity ratio is properly taken into account, as is necessary in order to analyse new thermal storage systems, particularly packed beds with water as a fluid, adsorbent materials[9–10], low-melting alloys and liquid–metal storage (studied for fast-breeder reactors and solar power plants with a central receiver[11]).

Several simplifying assumptions, including neglecting axial conduction in the solid, are as usual adopted in the analytical treatment of such two-phase models. However, the rigorous solution proposed improves understanding of the physical problem, and at the same time can provide a useful means of checking the predictions of computed solutions of less idealized systems.

Mathematical model and general solution

The mathematical approach is based on a thermal energy balance for either the fluid and the solid phase, and provides a set of two coupled partial differential equations[12]:

$$\rho_f c_f \varepsilon \frac{\partial \theta_f}{\partial \eta} + \rho_f c_f \varepsilon u \frac{\partial \theta_f}{\partial \xi} = ha(\theta_s - \theta_f) \qquad (1a)$$

$$\rho_s c_s (1-\varepsilon) \frac{\partial \theta_s}{\partial \eta} = ha(\theta_f - \theta_s) \qquad (1b)$$

Equations (1) entail the following customary assumptions: fluid in plug flow; constant physical parameters; negligible losses to the surroundings

* Laboratorio di Ingegneria Nucleare, Università di Bologna
† Istituto di Fisica Tecnica, Facoltà di Ingegneria, Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy
Received 4 December 1981 and accepted for publication on 26 April 1982

and negligible radiation effects; negligible heat conduction in the direction of fluid flow; and negligible temperature gradients normal to the direction of fluid flow.

Scaling the space and time coordinates according to a fluid characteristic length x and a solid characteristic time t, the non-dimensionalised variables arise:

$$x = \frac{ha}{\rho_f c_f \varepsilon u}\xi \qquad t = \frac{ha}{\rho_s c_s (1-\varepsilon)}\eta \tag{2}$$

together with the dimensionless parameter:

$$z = \frac{\rho_f c_f \varepsilon}{\rho_s c_s (1-\varepsilon)} \tag{3}$$

which represents the fluid-to-solid heat capacity ratio and can characterise significantly the dynamic response of storage units made by different materials. By introducing the dimensionless temperatures:

$$T_f = \frac{\theta_f - \theta_1}{\theta_2} \qquad T_s = \frac{\theta_s - \theta_1}{\theta_2} \tag{4}$$

where the typical values θ_1 and θ_2 will be specified later on; the coupled differential equations take the form

$$z\frac{\partial T_f}{\partial t} + \frac{\partial T_f}{\partial x} = T_s - T_f \tag{5a}$$

$$\frac{\partial T_s}{\partial t} = T_f - T_s \tag{5b}$$

Because of the operating conditions mentioned earlier it is assumed that the temperature of the fluid entering the storage unit is subjected to periodic variations, which in practice are due to intermittent energy supply and/or periodic utilisation demand. From the nature of the periodic phenomenon, the time variable has to be considered as ranging from $-\infty$ to $+\infty$, and initial conditions are replaced by the requirement of a continuous 'steady-state' repetitive operation of the unit. The inlet fluid temperature will be given by a known periodic function $\theta_f(0, t)$, which yields an inlet condition $T_f(0, t) = g(t)$, where g is the dimensionless form of $\theta_f(0, t)$ according to Eq (4).

The exact solution of the problem can be carried out by an exponential Fourier transform of parameter ω with respect to the t variable. The transformed equations are:

$$\frac{\partial \tilde{T}_f}{\partial x} + (1 + i\omega z)\tilde{T}_f(x, \omega) = \tilde{T}_s(x, \omega) \tag{6a}$$

$$(1 + i\omega)\tilde{T}_s(x, \omega) = \tilde{T}_f(x, \omega) \tag{6b}$$

and are readily solved by:

$$\tilde{T}_f(x, \omega) = \tilde{g}(\omega) \exp\left[-\left(1 + i\omega z - \frac{1}{1 + i\omega}\right)x\right] \tag{7a}$$

$$\tilde{T}_s(x, \omega) = \frac{\tilde{g}(\omega)}{1 + i\omega} \exp\left[-\left(1 + i\omega z - \frac{1}{1 + i\omega}\right)x\right] \tag{7b}$$

Transforming back to the original variable t yields, by way of the convolution theorem:

$$T_f(x, t) = \int_{-\infty}^{+\infty} T_f^G(x, t - t')\, g(t')\, dt' \tag{8a}$$

$$T_s(x, t) = \int_{-\infty}^{+\infty} T_s^G(x, t - t')\, g(t')\, dt' \tag{8b}$$

where the Green's functions[13] T_f^G, T_s^G are given by:

$$T_f^G(x, t) = \delta(t - zx)\, e^{-x} + U(t - zx)\left(\frac{x}{t - zx}\right)^{1/2} \times e^{-x-(t-zx)}\, I_1[2x^{1/2}(t - zx)^{1/2}] \tag{9a}$$

$$T_s^G(x, t) = U(t - zx)\, e^{-x-(t-zx)}\, I_0[2x^{1/2}(t - zx)^{1/2}] \tag{9b}$$

The physical expectation that $g(t')$ may affect T_g and T_s at x, t only for $t \geqslant t' + zx$ (corresponding to $\eta \geqslant \eta' + \xi/u$) is fulfilled, since Eqs (9) imply that the actual upper limit of integration in Eqs (8) is simply $(t - zx)$. The influence of z on the system response is now evident, as well as the actual dependence of the solution on the characteristics of Eq (5), namely x and $(t - zx)$ only.

The periodic nature of $\theta_f(0, t)$ can be exploited. It seems convenient to choose for θ_1 the minimum of θ_f over a period (denoted below by $2\pi/\omega_0$), so that $T_f = 0$ corresponds to 'cold' fluid. The total excursion of θ_f over a period (unit excursion on the T scale) is taken as θ_2. The inlet datum can then be Fourier-analysed as:

$$g(t) = \sum_{n=0}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \tag{10}$$

Nomenclature

a	Fluid–solid contact area per storage unit volume, m^{-1}
c	Specific heat, $J\,kg^{-1}\,°C^{-1}$
h	Fluid–solid heat transfer coefficient, $W\,m^{-2}\,°C^{-1}$
I_n	First-kind modified Bessel functions of order n
t	Dimensionless time
T	Dimensionless temperature
u	Mean fluid flow velocity, $m\,s^{-1}$
T	Dimensionless temperature
U	Heaviside function
x	Dimensionless space coordinate
z	Fluid to solid heat capacity ratio
δ	Dirac function
δ_{nk}	Kronecker symbol
ε	Void fraction
η	Time, s
θ	Temperature, °C
ξ	Space coordinate, m
ρ	Density, $kg\,m^{-3}$
ω	Angular frequency

Subscripts

f	Fluid
s	Solid

Superscripts

$\tilde{}$	Fourier-transformed function

with the usual definition of the coefficients a_n and b_n. Now, if $g(t)$ is continuous, the Fourier series (Eq (10)) can be introduced into Eqs (8) and integrated term by term, to obtain the series representation for the solution:

$$T_f(x,t)=\sum_{n=0}^{\infty}[a_n C^{(f)}_{n\omega_0}(x,t)+b_n S^{(f)}_{n\omega_0}(x,t)] \quad (11a)$$

$$T_s(x,t)=\sum_{n=0}^{\infty}[a_n C^{(s)}_{n\omega_0}(x,t)+b_n S^{(s)}_{n\omega_0}(x,t)] \quad (11b)$$

where $C^{(f)}_{\omega}$ and $C^{(s)}_{\omega}$ ($S^{(f)}_{\omega}$ and $S^{(s)}_{\omega}$) denote the fluid and solid response, respectively, to the cosinusoidal (sinusoidal) input with angular frequency ω. Such responses can be determined analytically. With reference to the case:

$$g(t)=\sin(\omega_0 t)$$

Eq (5) may be solved via Eqs (6)–(9), bearing in mind that delta functions arise as Fourier transforms of sines and cosines. The solution is simply given by:

$$S^{(f)}_{\omega_0}(x,t)=\exp\left(-\frac{\omega_0^2}{1+\omega_0^2}x\right)\times\sin\left[\omega_0\left(t-zx-\frac{x}{1+\omega_0^2}\right)\right] \quad (12a)$$

$$S^{(s)}_{\omega_0}(x,t)=\frac{1}{1+\omega_0^2}\exp\left(-\frac{\omega_0^2}{1+\omega_0^2}x\right)\times\left\{\sin\left[\omega_0\left(t-zx-\frac{x}{1+\omega_0^2}\right)\right]-\omega_0\cos\left[\omega_0\left(t-zx-\frac{x}{1-\omega_0^2}\right)\right]\right\} \quad (12b)$$

and exhibits an exponential damping combined with a linearly increasing phase shift. The corresponding cosinusoidal case gives analogously:

$$C^{(f)}_{\omega_0}(x,t)=\exp\left(-\frac{\omega_0^2}{1+\omega_0^2}x\right)\times\cos\left[\omega_0\left(t-zx-\frac{x}{1+\omega_0^2}\right)\right] \quad (12c)$$

$$C^{(s)}_{\omega_0}(x,t)=\frac{1}{1+\omega_0^2}\exp\left(-\frac{\omega_0^2}{1+\omega_0^2}x\right)\times\left\{\cos\left[\omega_0\left(t-zx-\frac{x}{1+\omega_0^2}\right)\right]+\omega_0\sin\left[\omega_0\left(t-zx-\frac{x}{1+\omega_0^2}\right)\right]\right\} \quad (12d)$$

Consequently $S^{(f)}_0=S^{(s)}_0=0$ and $C^{(f)}_0=C^{(s)}_0=1$.

The response to any inlet condition $g(t)$ is thus given in full by Eqs (11) and (12), and only the expansion coefficients of the trigonometric Fourier series for g need be calculated.

Applications and results

Immediate consequences of the analytical expressions in the previous section are that T_f and T_s are both periodic with the same period $2\pi/\omega_0$ as g, for any x, and their average over the period is still a_0, as is clear from a physical standpoint, since losses to the surroundings have been disregarded. Furthermore, for any value of t:

$$\lim_{x\to\infty} T_f(x,t)=\lim_{x\to\infty} T_s(x,t)=a_0 \quad (13)$$

so that a sufficiently long bed is able to smooth out, as much as desired, all time variations of the power supply.

In the following the response of the storage system to some typical trend of the inlet datum $g(t)$ is examined for different values of the dimensionless parameters x, z, ω_0, which also depend on the particular kind of storage unit.

(a) For a sinusoidal input:

$$g(t)=\frac{1+\sin(\omega_0 t)}{2} \quad (14)$$

and then $a_0=\frac{1}{2}$, $b_1=\frac{1}{2}$, whereas all other a_n and b_n vanish. The response thus exhibits the occurrence of a cosinusoidal term in the solid phase.

(b) In the absence of energy supply over a half period, and for a sinusoidal supply over the active half period, the inlet fluid temperature takes the form:

$$g(t)=U\left(\frac{\pi}{2\omega_0}-|t|\right)\cos(\omega_0 t) \qquad -\frac{\pi}{\omega_0}\leq t\leq\frac{\pi}{\omega_0} \quad (15)$$

while the coefficients of the Fourier series are given by:

$$a_0=\frac{1}{\pi} \qquad a_1=\frac{1}{2} \qquad a_n=\frac{(-1)^{(n-2)/2}}{\pi}\,\frac{1+(-1)^n}{n^2-1}$$

$$n=2,3,\ldots, \qquad b_n=0 \qquad n=1,2,\ldots \quad (16)$$

Some of the results obtained are shown in Fig 1.

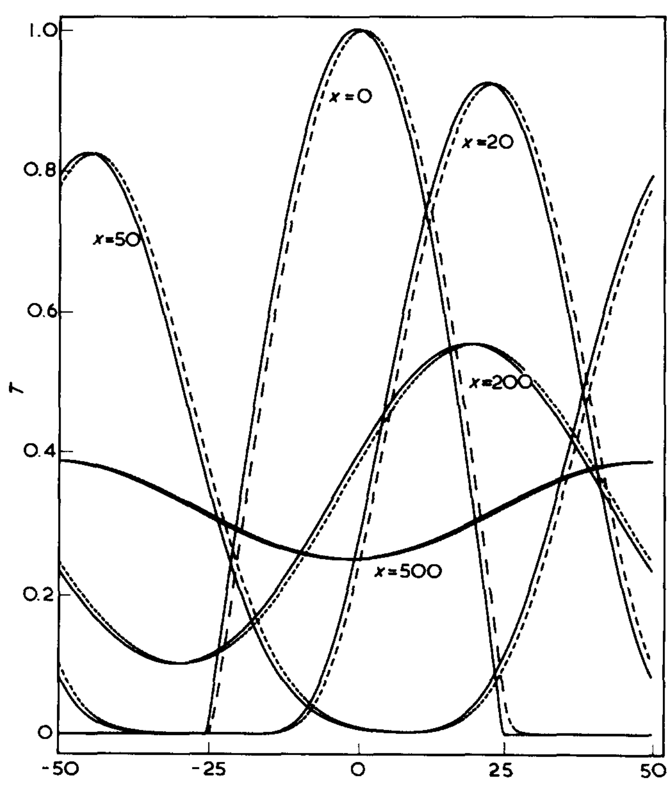

Fig 1 Fluid (solid line) and solid (dashed line) outlet temperatures versus time for different values of x *and for* z = 0.1*, in the case of Eq (15)*

(c) Another interesting trend of the inlet fluid temperature is given by a constant intermittent supply, with linear transition from and to the periods of no supply. The shape of the input is characterised by the slope of the linear transition, α, and the duration of constant supply, 2β, with

$$0 \leq \beta < \pi/\omega_0 \qquad 0 < 1/\alpha \leq (\pi/\omega_0) - \beta$$

and its analytic expression is:

$$g(t) = \begin{cases} 1 & |t| \leq \beta \\ 1 + \alpha\beta - \alpha|t| & \beta \leq |t| \leq \beta + 1/\alpha \\ 0 & \beta + 1/\alpha \leq |t| \leq \pi/\omega_0 \end{cases} \qquad (17)$$

A very fast switching between the two operations is described by the limit for $\alpha \to +\infty$. The Fourier coefficients can be evaluated analytically, thus:

$$a_0 = \frac{\omega_0}{\pi}\left(\beta + \frac{1}{2\alpha}\right)$$

$$a_n = \frac{4\alpha}{n^2 \pi \omega_0} \sin\left(\frac{n\omega_0}{2\alpha}\right) \times \sin\left[n\omega_0\left(\beta + \frac{1}{2\alpha}\right)\right] \qquad n = 1, 2, \ldots$$

$$b_n = 0 \qquad n = 1, 2, \ldots \qquad (18)$$

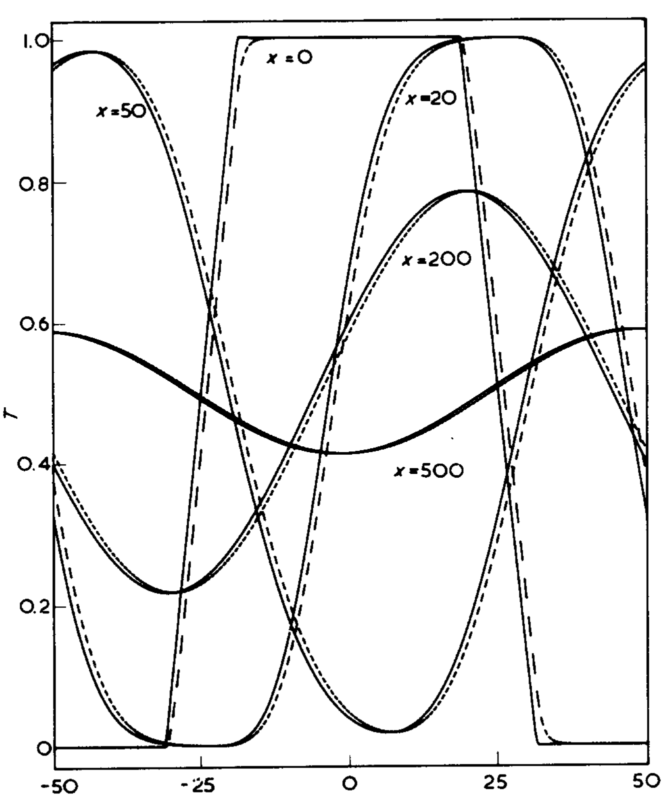

Fig 2 Fluid (solid line) and solid (dashed line) outlet temperatures versus time for different values of x *and for* z = *0.1, in the case of Eq (17), when* $\beta = \pi/2\omega_0 - 1/2\alpha = 3\pi/8\omega_0$

Some results for the symmetric case $\beta + (1/2\alpha) = \pi/2\omega_0$ are given in Fig 2.

Conclusions

In Figs 1 and 2 the temperatures T_f and T_s are plotted versus time over the period $(-\pi/\omega_0, \pi/\omega_0)$, with $\pi/\omega_0 = 50$, for different values of x ($x = 0, 20, 50, 200, 500$) but for a fixed value of z ($z = 0.1$). The effects of z on T_f and T_s are in fact apparent from Eqs (12): increasing z for a fixed x amounts simply to shifting the time variable by a constant time lag, and therefore it is sufficient to know the temperatures sought for a single value of z. It should also be noticed that the fluid temperature at $x = 0$ reproduces the inlet condition. Time variations at the inlet are clearly smoothed out as x increases, and a single sinusoidal-like term, with a period $2\pi/\omega_0$, will dominate for large x (compare Eqs (11) and (12)). Also, the locations of maxima and minima continuously increase with x, and there is a suitable bed length for which the warmest fluid at the outlet is obtained corresponding to the minimum power supply at the inlet.

References

1. **Kovach E. G.** *Thermal Energy Storage, 1976. Pergamon Press, Oxford*

2. **Hooper F. C.** The design of a solar heating system using seasonal heat storage. *Joint Conf. Am. Sec. ISES and SESCI, Winnipeg, Canada, Aug. 1976,* **8**, *260-261*

3. **Drew M. S. and Selvage R. B. G.** Solar heating system performance estimation using sinusoidal inputs. *Solar Energy, 1979,* **23**, *435-442*

4. **Drew M. S. and Selvage R. B. G.** Sizing procedure and economic optimization methodology for seasonal storage solar systems. *Solar Energy, 1980,* **25**, *79-83*

5. **Romie F. E.** Periodic thermal storage: the regenerator. *J. Heat Transfer, A.S.M.E. Ser. C, 1979,* **101**, *726-731*

6. **Chawla O. P. and Khandwala A. I.** Thermal performance of regenerators and waste heat recovery. *Int. J. Heat & Mass Transfer, 1981,* **24** *(II), 1793-1800*

7. **Clark J. A.** Thermal storage in rock beds. *Lecture Series 2, Von Karman Institute for Fluid Dynamics, Rhode St. Genese, 1980*

8. **Smith E. M.** General integral solution of the regenerator transient test equations for zero longitudinal conduction. *Int. J. Heat and Fluid Flow, 1979,* **1**, *71-75*

9. **Close D. J. and Pryor T. L.** The behaviour of adsorbent energy storage beds. *Solar Energy, 1976,* **18**, *287-292*

10. **Pryor T. L. and Close D. J.** Measurements of the behaviour of adsorbent energy storage beds. *Solar Energy, 1978,* **20**, *151-155*

11. **Pomeroy B. D.** Thermal energy storage in a packed bed of iron spheres with liquid sodium coolant. *Solar Energy, 1979,* **23**, *513-515*

12. **Spiga G. and Spiga M.** A rigorous solution to a heat transfer two phase model in porous media and packed beds. *Int. J. Heat & Mass Transfer, 1981,* **24**, *355-364*

13. **Spiga G. and Spiga M.** Analytical simulation in heat storage systems. *Int. Seminar 'Advancement in Heat Exchangers', I.C.H.M.T., Dubrovnik, Yugoslavia, Sept. 1981*